Question Number	Scheme		Marks
1. (a)	$\{\mathrm{P}(X>4)=\}\ 1-\mathrm{F}(4)$	$1-\mathrm{F}(4)$ seen or used	M1
	$\left\{=1-\dfrac{3}{5}\right\}=\dfrac{2}{5}$	$\dfrac{2}{5}$ or 0.4	A1
			[2]
(b)	$\mathrm{P}(3<X<a)=0.642$		
	$\mathrm{F}(a)-\mathrm{F}(3)=0.642$	$\mathrm{F}(a)-\mathrm{F}(3)=0.642$	M1 o.e.
	$\mathrm{F}(a)-\dfrac{1}{20}(3^2-4)=0.642\ \{\Rightarrow \mathrm{F}(a)=0.892\}$	Correct equation	A1 o.e.
	$\dfrac{1}{5}(2a-5)-\text{"}\dfrac{1}{20}(3^2-4)\text{"}=0.642\Rightarrow a=\ldots$	Solving this equation o.e., leading to $a=\ldots$ (or $x=\ldots$). Follow through their F(3)	dM1
	$\left\{\dfrac{1}{5}(2a-5)=0.892\ \Rightarrow\right\}\ a=4.73$	$a=4.73$ (or $x=4.73$)	A1 **cao**
			[4]
(b)	**<u>Alternative Method for Part (b)</u>**		
	$\displaystyle\int_3^4\left(\frac{1}{10}x\right)\{\mathrm{d}x\}$	Correct expression for finding the probability between $x=3$ and $x=4$	M1
	$\left\{=\left[\dfrac{x^2}{20}\right]_3^4\right\}=\dfrac{4^2}{20}-\dfrac{3^2}{20}\ \left\{=\dfrac{7}{20}\right\}$	Correct $\dfrac{4^2}{20}-\dfrac{3^2}{20}$, simplified or un-simplified.	A1
	$\displaystyle\int_3^4\left(\frac{1}{10}x\right)\{\mathrm{d}x\}+\int_4^a\left(\frac{2}{5}\right)\{\mathrm{d}x\}=0.642\Rightarrow a=\ldots$	Writes a correct equation and attempts to solve leading to $a=\ldots$ (or $x=\ldots$)	dM1
	$\left\{\dfrac{7}{20}+\dfrac{2}{5}a-\dfrac{8}{5}=0.642\ \Rightarrow\right\}\ a=4.73$	$a=4.73$ (or $x=4.73$)	A1 **cao**
(c)	$\mathrm{f}(x)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{20}(x^2-4)\right)=\dfrac{1}{10}x$ $\mathrm{f}(x)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{5}(2x-5)\right)=\dfrac{2}{5}$	Attempt at differentiation. **See notes.**	M1
		At least one of $\dfrac{1}{10}x$ or $\dfrac{2}{5}$	A1
		Both $\dfrac{1}{10}x$ and $\dfrac{2}{5}$	A1
	$\mathrm{f}(x)=\begin{cases}\dfrac{1}{10}x, & 2\leqslant x\leqslant 4\\ \dfrac{2}{5}, & 4<x\leqslant 5\\ 0, & \text{otherwise}\end{cases}$	**This mark is dependent on M1** All three lines with limits correctly followed through from their $\mathrm{F}'(x)$	dB1ft
			[4]
			10

2. The time, in minutes, spent waiting for a call to a call centre to be answered is modelled by the random variable T with probability density function

$$\mathrm{f}(t) = \begin{cases} \frac{1}{192}(t^3 - 48t + 128) & 0 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find, in minutes and seconds, the mean waiting time.

(3)

(b) Show that $\mathrm{P}(1 < T < 3) = \frac{7}{16}$

(3)

A supervisor randomly selects 256 calls to the call centre.

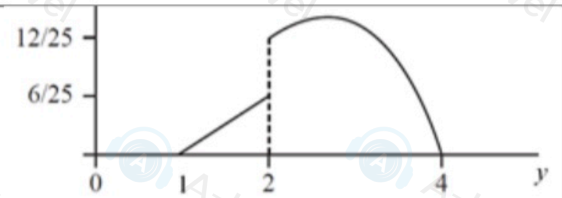
(c) Use a suitable approximation to find the probability that more than 125 of these calls take between 1 and 3 minutes to be answered.

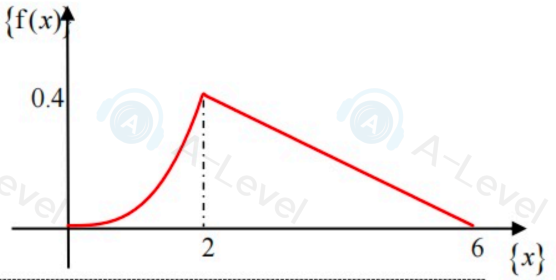
(5)

Question Number	Scheme	Marks
6 (a)	$\int_1^7 (ax^2 + bx)\mathrm{d}x[=1]$	M1
	$\left[\frac{ax^3}{3} + \frac{bx^2}{2}\right]_1^7 [=1]$	A1
	$\left(\frac{a(7^3)}{3} + \frac{b(7^2)}{2}\right) - \left(\frac{a}{3} + \frac{b}{2}\right) = 1$	dM1
	$114a + 24b = 1$ *	A1cso
		(4)
(b)	$114(\frac{1}{90}) + 24b = 1 \Rightarrow b = -\frac{1}{90}$	B1
	$[\mathrm{E}(X) =]\int_1^7 (\frac{1}{90}x^3 +' b' x^2)\mathrm{d}x$	M1
	$\left[\frac{x^4}{360} - \frac{x^3}{270}\right]_1^7, = \left(\frac{7^4}{360} - \frac{7^3}{270}\right) - \left(\frac{1}{360} - \frac{1}{270}\right) = 5.4$	A1ft, A1oe
		(4)
(c)	$\int_1^x (\frac{1}{90}t^2 - \frac{1}{90}t)\mathrm{d}t = \left[\frac{t^3}{270} - \frac{t^2}{180}\right]_1^x$ **or** $\int (\frac{1}{90}x^2 - \frac{1}{90}x)\mathrm{d}x = \frac{x^3}{270} - \frac{x^2}{180} + c$ with $\mathrm{F}(1) = 0$ or $\mathrm{F}(7) = 1$	M1
	$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ \frac{x^3}{270} - \frac{x^2}{180} + \frac{1}{540} & 1 \le x \le 7 \\ 1 & x > 7 \end{cases}$	A1 B1
		(3)
(d)	$\mathrm{P}(X >' 5.4') = 1 - \mathrm{F}('5.4') = 1 - 0.42305... = 0.5769....$ awrt **<u>0.577</u>**	M1 A1
		(2)
(e)	Since (d)>0.5, [the mean is less than the median] therefore negative (skew).	M1, A1
		(2)
		Total (15)

Question Number	Scheme	Marks
5(a)	F(6) = 1	
	$4k(12-7)=1$	M1
	$k=\frac{1}{20}$	A1
	$\alpha^2-2\alpha-3=4(2\alpha-7)$	M1
	$\alpha^2-10\alpha+25=0$	
	$(\alpha-5)^2=0$	
	$\alpha=5$	A1cao
	$\text{P}(4.5<X\le<5.5)=\text{F}(5.5)-\text{F}(4.5)$	M1
	$=4\times\frac{1}{20}\times(11-7)-\frac{1}{20}\times(4.5^2-9-3)$	dM1
	$=\frac{31}{80}$ or 0.3875 or awrt 0.388	A1
		(7)
(b)	$\text{f}(y)=\begin{cases}\frac{1}{20}(2y-2) & 3\le y\le 5\\ \frac{2}{5} & 5<y\le 6\\ 0 & \text{otherwise}\end{cases}$	M1 A1ft A1
		(3)
	Notes	**Total 10**

Question Number	Scheme	Marks
5 (a)	$\int_1^x \frac{1}{4}(3-t)\,\mathrm{d}t = \frac{1}{4}\left[3t-\frac{t^2}{2}\right]_1^x$ or $\int \frac{1}{4}(3-x)\,\mathrm{d}x = \frac{1}{4}\left[3x-\frac{x^2}{2}\right]+C$	M1
	$\frac{1}{4}\left[\left(3x-\frac{x^2}{2}\right)-\left(3-\frac{1}{2}\right)\right]$ or $\frac{1}{4}\left[3(1)-\frac{(1)^2}{2}\right]+C=0$ and $C=-\frac{5}{8}$ Leading to $\frac{1}{4}\left(3x-\frac{x^2}{2}\right)-\frac{5}{8}$ [for $1 \leqslant x \leqslant 2$]*	A1*
		(2)
(b)	$\int_2^x \frac{1}{4}\mathrm{d}t+\mathrm{F}(2)$ or $\int \frac{1}{4}\mathrm{d}x$ and using $+c$ with $\mathrm{F}(2)=\frac{3}{8}$ or $0.25(x-2)+\mathrm{F}(2)$	M1
	$\int_3^x \frac{1}{4}(t-2)\,\mathrm{d}t+\mathrm{F}(3)$ or $\int \frac{1}{4}(x-2)\,\mathrm{d}x$ and using $+c$ with either $\mathrm{F}(3)=\frac{5}{8}$ or $\mathrm{F}(4)=1$	M1
	$\mathrm{F}(x)=\begin{cases} 0 & x<1 \\ \frac{1}{4}\left(3x-\frac{x^2}{2}\right)-\frac{5}{8} & 1 \leqslant x \leqslant 2 \\ \frac{1}{8}(2x-1) & 2<x \leqslant 3 \\ \frac{1}{4}\left(\frac{x^2}{2}-2x\right)+1 & 3<x \leqslant 4 \\ 1 & x>4 \end{cases}$	A1 A1 B1
		(5)
(c)	$\mathrm{P}(1.2<X<3.1)=\mathrm{F}(3.1)-\mathrm{F}(1.2)$	
	$\left(\frac{1}{4}\left(\frac{(3.1)^2}{2}-2(3.1)\right)+1\right)-\left(\frac{1}{4}\left(3(1.2)-\frac{(1.2)^2}{2}\right)-\frac{5}{8}\right)=\frac{89}{160}$ awrt 0.556	M1 A1
		(2)
	Notes	**Total 9**

Question Number	Scheme	Marks
3. (a)		M1 A1 (2)
(b)	$\dfrac{\mathrm{d}\left(\frac{3}{50}\left(4y^2-y^3\right)\right)}{\mathrm{d}y}=\frac{3}{50}\left(8y-3y^2\right)$	M1
	$\frac{3}{50}\left(8y-3y^2\right)=0$; $\underline{y=\frac{8}{3}}$ oe	M1; A1 (3)
(c)	$\mathrm{E}(Y^2)=\int_1^2\left(\frac{6}{25}y^3-\frac{6}{25}y^2\right)\mathrm{d}y+\int_2^4\left(\frac{12}{50}y^4-\frac{3}{50}y^5\right)\mathrm{d}y$	M1
	$=\left[\frac{6}{100}y^4-\frac{6}{75}y^3\right]_1^2+\left[\frac{12}{250}y^5-\frac{3}{300}y^6\right]_2^4$	A1
	$=\left[\left(\frac{8}{25}\right)-\left(-\frac{1}{50}\right)\right]+\left[\left(\frac{1024}{125}\right)-\left(\frac{112}{125}\right)\right]$; $=\underline{\frac{1909}{250}}$ or $\underline{7.636}$ or $\underline{7.64}$	dM1; A1 (4)
(d)	$\mathrm{Var}(Y)=\text{"}\frac{1909}{250}\text{"}-2.696^2$	M1
	$=0.367584$ awrt $\underline{\mathbf{0.368}}$	A1 (2)
(e)	$\frac{1}{2}(y-1)\times\frac{6}{25}(y-1)=0.1$ <u>or</u> $\int_1^x\frac{6}{25}(y-1)\,\mathrm{d}y=0.1$	M1
	$\frac{1}{2}(y-1)\times\frac{6}{25}(y-1)=0.1$ <u>or</u> $\frac{6}{25}\left[\left(\frac{x^2}{2}-x\right)+\frac{1}{2}\right]=0.1$ <u>or</u> $\frac{6}{50}(x-1)^2=0.1$	A1
	$(y-1)^2=\frac{5}{6}$ <u>or</u> $y=1\pm\sqrt{\frac{5}{6}}$; $y=1.9128...$ awrt $\underline{\mathbf{1.91}}$	dM1; A1 (4)
		Total 15

Question Number	Scheme	Marks
7. **(a)**	 Correct shape with correct curvature and straight line with negative gradient. Must start and end on the x-axis.	B1
	2, 6 and 0.4 labelled in the correct place	B1
		[2]
(b)	$\{\text{Mode} = \}\ 2$ **2**	B1
		[1]
(c)	$\{P(X>2)=\}\ \int_2^6 \frac{1}{10}(6-x)\,dx$ **or** $\frac{1}{2}(6-2)(0.4)$ **or** $1-\int_0^2 \frac{1}{20}x^3\,dx$	M1
	$= 0.8$ **0.8**	A1* **cso**
		[2]
(d)	$\frac{1}{80}x^4,\ 0 \le x \le 2$	B1
	$\int_0^2 \frac{1}{20}t^3\,dt + \int_2^x \frac{1}{10}(6-t)\,dt = 0.2 + \frac{1}{10}\left[6t - \frac{1}{2}t^2\right]_2^x$ **or** $\int \frac{1}{10}(6-x)\,dx = \frac{1}{10}(6x - \frac{1}{2}x^2) + c$ or $-\frac{1}{20}(6-x)^2 + d$ with $F(2)=0.2$ or $F(6)=1$	M1
	$F(x) = \begin{cases} 0 & x<0 \\ \frac{1}{80}x^4 & 0 \le x \le 2 \\ \frac{1}{10}(6x - \frac{1}{2}x^2 - 8)\text{ o.e.} & 2 < x < 6 \\ 1 & x > 6 \end{cases}$ Condone $\le$ for $<$ (etc.) throughout part (d) and vice versa	A1 B1
		[4]
(e)	$\left\{P(X<a \mid X>2) = \frac{5}{8} \Rightarrow F(a) = \right\}\ \frac{5}{8}(0.8) + 0.2;\ = 0.7$ **0.7**	M1A1
		[2]
(f)	$\frac{1}{10}\left(6a - \frac{1}{2}a^2 - 8\right) = \frac{7}{10}$ **or** $\frac{1}{2}(6-a) \leftrightarrow \frac{1}{10}(6-a) = 0.3$	M1
	$\left\{a^2 - 12a + 30 = 0 \Rightarrow\right\}\ a = \frac{12 \pm \sqrt{12^2 - 4(1)(30)}}{2}$	dM1
	$\{a = 3.5505102... \Rightarrow\}\ a = 3.55\ (3\text{sf})$ awrt **3.55** only	A1
		[3]
		14

Question Number	Scheme	Marks
5(a)	$\mathrm{E}(T^2)=\int_0^3 \frac{1}{50}\left(18t^2-2t^3\right)\,\mathrm{d}t+\int_3^5 \frac{1}{20}t^2\mathrm{d}t$	M1
	$=\left[\frac{1}{50}\left(6t^3-\frac{t^4}{2}\right)\right]_0^3+\left[\frac{t^3}{60}\right]_3^5$ or $=\left[\frac{3}{25}t^3-\frac{t^4}{100}\right]_0^3+\left[\frac{t^3}{60}\right]_3^5$ oe	A1
	$=\frac{1}{50}\left(6\times3^3-\frac{3^4}{2}\right)+\left(\frac{125}{60}-\frac{27}{60}\right)$ or $=\frac{1}{50}\left(162-\frac{81}{2}\right)+\left(\frac{25}{12}-\frac{9}{20}\right)$ oe	M1d
	$=\frac{1219}{300}=4.063...$	
	$\mathrm{Var}(T)=\text{"}4.063...\text{"}-(1.66)^2$	M1
	$=1.3077...$ awrt 1.31	A1
		(5)
(b)	$\int_3^t \frac{1}{20}\mathrm{d}x+C$ where $C=0.9$ or $\int_0^3 \frac{1}{50}(18-2t)\,\mathrm{d}t$ **or** using F(5) =1 to find C	M1
	$[\mathrm{F}(t)=]\begin{cases} 0 & t<0 \\ \frac{1}{50}\left(18t-t^2\right) \text{ or } 1.62-\frac{(18-2t)^2}{200} & 0\le t\le 3 \\ \frac{1}{20}t+0.75 & 3<t\le 5 \\ 1 & t>5 \end{cases}$	B1 A1 A1
		(4)
(c)	$\mathrm{P}(T>2)=1-\text{"}\frac{1}{50}\left(18\times2-2^2\right)\text{"}$ or $1-\int_0^2 \frac{1}{50}(18-2t)\,\mathrm{d}t$	M1
	$=\frac{9}{25}$ or 0.36	A1
		(2)
(d)	$\mathrm{P}(0<T<3.66)=\mathrm{F}(3.66)$	M1
	$=0.933$	A1
		(2)
	Notes	**Total 13**

Question	Scheme	Marks
3.(a)		B1 (shape) B1 (domain)
	This is not a valid probability density function since $\mathrm{f}(x) < 0$ for $x > 3$	B1 (3)
(b)	$[\mathrm{g}(y) = k(12y - y^3)]$	
	$[\mathrm{g}'(y)] = k(12 - 3y^2)$	M1
	$12 - 3y^2 = 0$	M1
	$y = 2$	A1 (3)
(c)	$\int k(12y - y^3)\mathrm{d}y = k\left[6y^2 - \frac{y^4}{4}\right]_1^3$	M1
	$k\left(6 \times 3^2 - \frac{3^4}{4} - 6 + \frac{1}{4}\right) = 1$	M1
	$k = \frac{1}{28}$	A1 (3)
(d)	$\int_1^m \frac{1}{28}(12y - y^3)\mathrm{d}y = \frac{1}{28}\left(6 \times m^2 - \frac{m^4}{4} - \left(6 \times 1^2 - \frac{1^4}{4}\right)\right) = 0.5$	M1
	$m^4 - 24m^2 + 79 = 0 \rightarrow m = 1.98437...$ awrt **<u>1.98</u>**	M1 A1 (3)
(e)	Median ≈ Mode, therefore there is no skew.	M1A1ft (2)
		Total 14