

6. (i) (a) State the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

(1)

A factory produces tyres for bicycles and 0.25% of the tyres produced are defective.

A company orders 3000 tyres from the factory.

- (b) Find, using a Poisson approximation, the probability that there are more than 7 defective tyres in the company's order.

(3)

- (ii) At the company 40% of employees are known to cycle to work. A random sample of 150 employees is taken. The random variable C represents the number of employees in the sample who cycle to work.

- (a) Describe a suitable sampling frame that can be used to take this sample.

(1)

- (b) Explain what you understand by the sampling distribution of C

(1)

Louis uses a normal approximation to calculate the probability that at most α employees in the sample cycle to work. He forgets to use a continuity correction and obtains the incorrect probability 0.0668

Find, showing all stages of your working,

- (c) the value of α

(4)

- (d) the correct probability.

(2)

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4. (i) The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that

- $P(X > 27) = \frac{3}{4}$
- $\text{Var}(X) = 300$

- (a) find the value of a and the value of b

(3)

Given also that

$$4 \times P(X < k - 10) = P(X > k + 20)$$

- (b) find the value of k

(2)

- (ii) A piece of wire of length 42 cm is cut into 2 pieces at a random point.

Each of the two pieces of the wire is bent to form the outline of a square.

Find the probability that the side length of the larger square minus the side length of the smaller square will be greater than 2 cm.

(4)

7. (i) The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that $P(5 < X < 13) = \frac{1}{5}$ and $E(X) = 9$, find $P(3X > a + b)$

(3)

- (ii) The continuous random variable Y is uniformly distributed over the interval $[1, c]$

Given that $\text{Var}(Y) = 0.48$, find the exact value of $E(Y^2)$

(4)

- (iii) A wire of length 20 cm is cut into 2 pieces at a random point.

The longest piece of wire is then cut into 2 pieces, equal in length, giving 3 pieces of wire altogether.

Find the probability that the length of the shortest piece of wire is less than 6 cm.

(5)

- (e) State the actual significance level of this test. (1)

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3. Every morning Navtej travels from home to work. Navtej leaves home at a random time between 08:00 and 08:15

- It always takes Navtej 3 minutes to walk to the bus stop
- Buses run every 15 minutes and Navtej catches the first bus that arrives
- Once Navtej has caught the bus it always takes a further 29 minutes for Navtej to reach work

The total time, T minutes, for Navtej's journey from home to work is modelled by a continuous uniform distribution over the interval $[\alpha, \beta]$

(a) (i) Show that $\alpha = 32$

(ii) Show that $\beta = 47$

(2)

(b) State fully the probability density function for this distribution.

(2)

(c) Find the value of

(i) $E(T)$

(ii) $\text{Var}(T)$

(3)

(d) Find the probability that the time for Navtej's journey is within 5 minutes of 35 minutes.

(2)