Leave blank

1. A continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{20}(x^2 - 4) & 2 \leqslant x \leqslant 4 \\ \frac{1}{5}(2x - 5) & 4 < x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

(a) Calculate $\mathrm{P}(X > 4)$

(2)

(b) Find the value of a such that $\mathrm{P}(3 < X < a) = 0.642$

(4)

(c) Find the probability density function of X, specifying it for all values of x.

(4)

2. The time, in minutes, spent waiting for a call to a call centre to be answered is modelled by the random variable T with probability density function

$$\mathrm{f}(t) = \begin{cases} \dfrac{1}{192}\left(t^3 - 48t + 128\right) & 0 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find, in minutes and seconds, the mean waiting time.

(3)

(b) Show that $\mathrm{P}(1 < T < 3) = \dfrac{7}{16}$

(3)

A supervisor randomly selects 256 calls to the call centre.

(c) Use a suitable approximation to find the probability that more than 125 of these calls take between 1 and 3 minutes to be answered.

(5)

6. A continuous random variable X has probability density function

$$f(x) = \begin{cases} ax^2 + bx & 1 \leqslant x \leqslant 7 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.

(a) Show that $114a + 24b = 1$

(4)

Given that $a = \dfrac{1}{90}$

(b) use algebraic integration to find $\mathrm{E}(X)$

(4)

(c) find the cumulative distribution function of X, specifying it for all values of x

(3)

(d) find $\mathrm{P}(X > \mathrm{E}(X))$

(2)

(e) use your answer to part (d) to describe the skewness of the distribution.

(2)

Leave blank

5. The continuous random variable Y has cumulative distribution function $\mathrm{F}(y)$ given by

$$\mathrm{F}(y) = \begin{cases} 0 & y < 3 \\ k(y^2 - 2y - 3) & 3 \leqslant y \leqslant \alpha \\ 4k(2y - 7) & \alpha < y \leqslant 6 \\ 1 & y > 6 \end{cases}$$

where k and α are constants.

(a) Find $\mathrm{P}(4.5 < Y \leqslant 5.5)$

(7)

(b) Find the probability density function $\mathrm{f}(y)$

(3)

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5. The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(3 - x) & 1 \leqslant x \leqslant 2 \\ \frac{1}{4} & 2 < x \leqslant 3 \\ \frac{1}{4}(x - 2) & 3 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

The cumulative distribution function of X is $F(x)$

(a) Show that $F(x) = \frac{1}{4}\left(3x - \frac{x^2}{2}\right) - \frac{5}{8}$ for $1 \leqslant x \leqslant 2$

(2)

(b) Find $F(x)$ for all values of x

(5)

(c) Find $P(1.2 < X < 3.1)$

(2)

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3. The continuous random variable Y has the following probability density function

$$f(y) = \begin{cases} \frac{6}{25}(y - 1) & 1 \leqslant y < 2 \\ \frac{3}{50}(4y^2 - y^3) & 2 \leqslant y < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(y)$

(2)

(b) Find the mode of Y

(3)

(c) Use algebraic integration to calculate $E(Y^2)$

(4)

Given that $E(Y) = 2.696$

(d) find $\text{Var}(Y)$

(2)

(e) Find the value of y for which $P(Y \geqslant y) = 0.9$
Give your answer to 3 significant figures.

(4)

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7. The continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{20}x^3 & 0 \leqslant x \leqslant 2 \\[2ex] \dfrac{1}{10}(6 - x) & 2 < x \leqslant 6 \\[2ex] 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $\mathrm{f}(x)$ for all values of x.

(2)

(b) Write down the mode of X.

(1)

(c) Show that $\mathrm{P}(X > 2) = 0.8$

(2)

(d) Define fully the cumulative distribution function $\mathrm{F}(x)$.

(4)

Given that $\mathrm{P}(X < a \mid X > 2) = \dfrac{5}{8}$

(e) find the value of $\mathrm{F}(a)$.

(2)

(f) Hence, or otherwise, find the value of a. Give your answer to 3 significant figures.

(3)

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5. The waiting time, T minutes, of a customer to be served in a local post office has probability density function

$$f(t) = \begin{cases} \dfrac{1}{50}(18 - 2t) & 0 \leqslant t \leqslant 3 \\ \dfrac{1}{20} & 3 < t \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

Given that the mean number of minutes a customer waits to be served is 1.66

(a) use algebraic integration to find $\text{Var}(T)$, giving your answer to 3 significant figures.
(5)

(b) Find the cumulative distribution function $F(t)$ for all values of t.
(4)

(c) Calculate the probability that a randomly chosen customer's waiting time will be more than 2 minutes.
(2)

(d) Calculate $P([E(T) - 2] < T < [E(T) + 2])$
(2)

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3. The function $f(x)$ is defined as

$$f(x) = \begin{cases} \frac{1}{9}(x+5)(3-x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

Albert believes that $f(x)$ is a valid probability density function.

(a) Sketch $f(x)$ and comment on Albert's belief.

(3)

The continuous random variable Y has probability density function given by

$$g(y) = \begin{cases} ky(12 - y^2) & 1 \leqslant y \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(b) Use calculus to find the mode of Y

(3)

(c) Use algebraic integration to find the value of k

(3)

(d) Find the median of Y giving your answer to 3 significant figures.

(3)

(e) Describe the skewness of the distribution of Y giving a reason for your answer.

(2)