Leave blank

6. One side of a square is measured to the nearest centimetre and this measurement is multiplied by 4 to estimate the perimeter of the square. The random variable, W cm, represents the estimated perimeter of the square minus the true perimeter of the square.

W is uniformly distributed over the interval $[a, b]$

(a) Explain why $a = -2$ and $b = 2$

(1)

The standard deviation of W is σ

(b) (i) Find σ

(ii) Find the probability that the estimated perimeter of the square is within σ of the true perimeter of the square.

(4)

One side of each of 100 squares are now measured. Using a suitable approximation,

(c) find the probability that W is greater than 1.9 for at least 5 of these squares.

(4)

Leave blank

3. The continuous random variable Y is such that $Y \sim U[a, b]$, where a and b are constants.

The mean of Y is 6 and the variance of Y is 3

(a) Find the value of a and the value of b.

(3)

(b) Find the probability that Y is more than 1 standard deviation above the mean.

(2)

(c) Define fully the cumulative distribution function of Y.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE

3. A point is to be randomly plotted on the x-axis, where the units are measured in cm.

The random variable R represents the x coordinate of the point on the x-axis and R is uniformly distributed over the interval $[-5, 19]$

A negative value indicates that the point is to the left of the origin and a positive value indicates that the point is to the right of the origin.

(a) Find the exact probability that the point is plotted to the right of the origin.

(1)

(b) Find the exact probability that the point is plotted more than 3.5 cm away from the origin.

(2)

(c) Sketch the cumulative distribution function of R

(2)

Three independent points with x coordinates R_1, R_2 and R_3 are plotted on the x-axis.

(d) Find the exact probability that

(i) all three points are more than 10 cm from the origin

(3)

(ii) the point furthest from the origin is more than 10 cm from the origin.

(2)

Leave blank

2. The continuous random variable X is uniformly distributed over the interval $[\alpha, \beta]$ where $\beta > \alpha$

Given that $E(X) = 8$

(a) write down an equation involving α and β

(1)

Given also that $P(X \leqslant 13) = 0.7$

(b) find the value of α and the value of β

(3)

(c) find $\text{Var}(X)$

(1)

(d) find $P(5 \leqslant X \leqslant 35)$

(2)

6. The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that

- $P(2 < X < 5) = \frac{3}{16}$
- $E(X) = 6$

(a) find the value of a and the value of b

(3)

(b) find the value of the constant c such that $E(cX + 1) = 3$

(2)

(c) find the exact value of $\text{Var}(X)$

(1)

(d) find the exact value of $E(X^2)$

(2)

(e) find $P\left(\frac{3}{2}X - b > a\right)$

(2)

DO NOT WRITE IN THIS AREA DO N

Leave blank

7 The sides of a square are each of length L cm and its area is A cm^2

Given that A is uniformly distributed on the interval $[10, 30]$

(a) find $P(L \geqslant 4.5)$

(2)

(b) find $\text{Var}(L)$

(6)

DO NOT WRITE IN T

Leave blank

3. A task was designed to take a person 25 minutes to complete. The random variable T represents the time, in minutes, it takes to complete the task and is uniformly distributed over the interval $[25 - k, 25 + 3k]$ where $k > 0$

(a) Find $P(25 - 0.5k < T < 25 + 2.5k)$

(1)

Given that $E(T^2) = 918.76$

(b) find the value of k.

(6)

The task is to be completed by 50 people. Assuming that people complete the task independently,

(c) calculate the probability that at least 20 of these people complete the task in less than 25 minutes.

(3)

DO NOT WRITE IN THIS AREA

Leave blank

3. Albert uses scales in his kitchen to weigh some fruit.

The random variable D represents, in grams, the weight of the fruit given by the scales **minus** the true weight of the fruit. The random variable D is uniformly distributed over the interval $[-2.5, 2.5]$

(a) Specify the probability density function of D

(2)

(b) Find the standard deviation of D

(2)

Albert weighs a banana on the scales.

(c) Write down the probability that the weight given by the scales equals the true weight of the banana.

(1)

(d) Find the probability that the weight given by the scales is within 1 gram of the banana's true weight.

(1)

Albert weighs 10 bananas on the scales, one at a time.

(e) Find the probability that the weight given by the scales is within 1 gram of the true weight for at least 6 of the bananas.

(3)

5. The random variable W has a continuous uniform distribution over the interval $[-6, a]$ where a is a constant.

Given that $\text{Var}(W) = 27$

(a) show that $a = 12$

(2)

Given that $\text{P}(W > b) = \frac{3}{5}$

(b) (i) find the value of b

(2)

(ii) find $\text{P}\left(-12 < W < \frac{b}{2}\right)$

(2)

A piece of wood AB has length 160 cm. The wood is cut at random into 2 pieces. Each of the pieces is then cut in half. The four pieces are used to form the sides of a rectangle.

(c) Calculate the probability that the area of the rectangle is greater than $975\,\text{cm}^2$

(4)