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4. The time, in thousands of hours, that a certain electrical component will last is modelled by the random variable X, with probability density function

$$f(x) = \begin{cases} \frac{3}{64}x^2(4-x) & 0 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

Using this model, find, by algebraic integration,

(a) the mean number of hours that a component will last,

(4)

(b) the standard deviation of X.

(4)

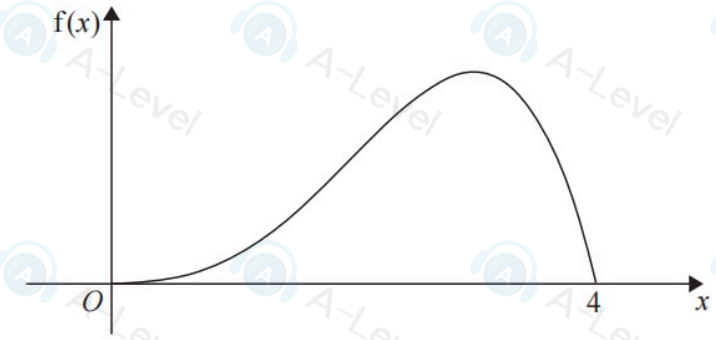

Figure 1

Figure 1 shows a sketch of the probability density function of the random variable X.

(c) Give a reason why the random variable X might be unsuitable as a model for the time, in thousands of hours, that these electrical components will last.

(1)

(d) Sketch a probability density function of a more realistic model.

(1)

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2. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4} & -\frac{1}{2} \leqslant x < \frac{1}{2} \\ 2x - \frac{3}{4} & \frac{1}{2} \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch the graph of $f(x)$

(2)

(b) By forming and solving an equation in k, show that $k = 1.25$

(4)

(c) Use calculus to find $E(X)$

(4)

(d) Calculate the interquartile range of X

(5)

5. The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(x^2 + a) & -1 < x \leqslant 2 \\ 3k & 2 < x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are constants.

Given that $E(X) = \frac{17}{12}$

(a) find the value of k and the value of a

(8)

(b) Write down the mode of X

(1)

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4. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{15}x^2(3-x) & 1 \leqslant x < 3 \\ \frac{3}{10}(x-3) & 3 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find $E(X^2)$

(4)

(b) Define fully the cumulative distribution function $F(x)$

(5)

(c) Find $P(2 < X < 4)$

(2)

(d) Find, to 3 significant figures, the value of k such that $P(X > k) = 0.2$

(3)

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2. The random variable $X \sim B(10, p)$

(a) (i) Write down an expression for $P(X = 3)$ in terms of p

(ii) Find the value of p such that $P(X = 3)$ is 16 times the value of $P(X = 7)$

(4)

The random variable $Y \sim Po(\lambda)$

(b) Find the value of λ such that $P(Y = 3)$ is 5 times the value of $P(Y = 5)$

(3)

The random variable $W \sim B(n, 0.4)$

(c) Find the value of n and the value of α such that W can be approximated by the normal distribution, $N(32, \alpha)$

(3)

4. The continuous random variable G has probability density function $f(g)$ given by

$$f(g) = \begin{cases} \frac{1}{15}(g+3) & -1 < g \leqslant 2 \\ \frac{3}{20} & 2 < g \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $f(g)$

(2)

(b) Find $P\left((1 \leqslant 2G \leqslant 6) \,|\, G \leqslant 2\right)$

(4)

The continuous random variable H is such that $E(H) = 12$ and $Var(H) = 2.4$

(c) Find $E\left(2H^2 + 3G + 3\right)$

Show your working clearly.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

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1. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{16}(x-1)^2 & 1 \leqslant x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

(a) Find $P(X > 4)$

(2)

(b) Find $P(X > 3 \,|\, 2 < X < 4)$

(3)

(c) Find the exact value of $E(X)$

(4)