5. The continuous random variable X is normally distributed with

$$X \sim \mathrm{N}(\mu, 5^2)$$

A random sample of 10 observations of X is taken and $\bar{X}$ denotes the sample mean.

(a) Show that a 90% confidence interval for μ, in terms of $\bar{x}$, is given by

$$(\bar{x} - 2.60, \bar{x} + 2.60)$$

(3)

The continuous random variable Y is normally distributed with

$$Y \sim \mathrm{N}(\mu, 3^2)$$

A random sample of 20 observations of Y are taken and $\bar{Y}$ denotes the sample mean.

(b) Find a 95% confidence interval for μ, in terms of $\bar{y}$

(3)

(c) Given that X and Y are independent,

(i) find the distribution of $\bar{X} - \bar{Y}$

(ii) calculate the probability that the two confidence intervals from part (a) and part (b) do not overlap.

(7)

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7. A company sells two types of board, short or long.

The weight of a short board, Skg, has distribution $\mathrm{N}(7.7, 0.01^2)$

The weight of a long board, Lkg, has distribution $\mathrm{N}(20, 0.02^2)$

A random sample of 3 short boards and 4 long boards is taken.

(a) Find the probability that the total weight of the 3 short boards and the 4 long boards is more than 103.15 kg

(4)

A random sample of 2 long boards is taken.

(b) Find the probability that the weights of these 2 long boards differ by more than 0.01 kg

(5)

A random sample of $(n + 1)$ short boards is taken, with weights $S_1, S_2, S_3, \ldots, S_{n+1}$

The random variable T is defined as

$$T = nS_1 - \sum_{r=2}^{n+1} S_r$$

Given that, to 4 decimal places, $\mathrm{P}(T > 2) = 0.0233$

(c) calculate the value of n

(6)

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7. The independent random variables X and Y are such that

$$X \sim N(30, 4.5^2) \text{ and } Y \sim N(20, 3.5^2)$$

The random variables X_1, X_2 and X_3 are independent and each has the same distribution as X. The random variables Y_1 and Y_2 are independent and each has the same distribution as Y.

Given that the random variable A is defined as

$$A = \frac{X_1 + X_2 + X_3 + Y_1 + Y_2}{5}$$

(a) find $P(A < 24)$

(6)

The random variable W is such that $W \sim N(\mu, 2.8^2)$

Given that $P(W - X < 4) = 0.1$ and that W and X are independent,

(b) find the value of μ, giving your answer to 3 significant figures.

(6)

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7. A company produces bricks.

The weight of a brick, B kg, is such that $B \sim N\left(1.96, \sqrt{0.003}^2\right)$

Two bricks are chosen at random.

(a) Find the probability that the difference in weight of the 2 bricks is greater than 0.1 kg

(5)

A random sample of n bricks is to be taken.

(b) Find the minimum sample size such that the probability of the sample mean being greater than 2 is less than 1%

(5)

The bricks are randomly selected and stacked on pallets.

The weight of an empty pallet, E kg, is such that $E \sim N\left(21.8, \sqrt{0.6}^2\right)$

The random variable M represents the total weight of a pallet stacked with 500 bricks.

The random variable T represents the total weight of a container of cement.

Given that T is independent of M and that $T \sim N\left(774, \sqrt{1.8}^2\right)$

(c) calculate $P(4T > 100 + 3M)$

(7)

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5. A dance studio has 800 dancers of which

452 are beginners
251 are intermediates
97 are professionals

(a) Explain in detail how a stratified sample of size 50 could be taken.

(3)

(b) State an advantage of stratified sampling rather than simple random sampling in this situation.

(1)

Independent random samples of 80 beginners and 60 intermediates are chosen. Each of these dancers is given an assessment score, x, based on the quality of their dancing. The results are summarised in the table below.

	$\bar{x}$	s^2	n
Beginners	31.7	57.3	80
Intermediates	36.9	38.1	60

The studio manager believes that the mean score of intermediates is more than 3 points greater than the mean score of beginners.

(c) Stating your hypotheses clearly and using a 5% level of significance, test whether or not these data support the studio manager's belief.

(7)

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6. The number of toasters sold by a shop each week may be modelled by a Poisson distribution with mean 4

A random sample of 35 weeks is taken and the mean number of toasters sold per week is found.

(a) Write down the approximate distribution for the mean number of toasters sold per week from a random sample of 35 weeks.

(2)

The number of kettles sold by the shop each week may be modelled by a Poisson distribution with mean λ

A random sample of 40 weeks is taken and the mean number of kettles sold per week is found. The width of the 99% confidence interval for λ is 2.6

(b) Find an estimate for λ

(4)

A second, independent random sample of 40 weeks is taken and a second 99% confidence interval for λ is found.

(c) Find the probability that only one of these two confidence intervals contains λ

(2)

7. Small containers and large containers are independently filled with fruit juice.

The amounts of fruit juice in small containers are normally distributed with mean 180 ml and standard deviation 4.5 ml

The amounts of fruit juice in large containers are normally distributed with mean 330 ml and standard deviation 6.7 ml

The random variable W represents the total amount of fruit juice in a random sample of 2 small containers minus the amount of fruit juice in 1 randomly selected large container.

$W \sim N(a, b)$ where a and b are positive constants.

(a) Find the value of a and the value of b

(3)

(b) Find the probability that a randomly chosen large container of fruit juice contains more than 1.8 times the amount of fruit juice in a randomly chosen small container.

(5)

A random sample of 3 small containers of fruit juice is taken.

(c) Find the probability that the first container of fruit juice in this sample contains at least 5 ml more than the mean amount of fruit juice in all 3 small containers.

(6)

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